Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Approximate Formula of Weak Oblique Shock Wave Angle"

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Introduction

IN Ref. 1 the authors describe two approximate formulas for evaluation of the wave angle β of an oblique shock wave in terms of the deflection angle θ and the upstream Mach number M_1. These formulas are only valid for a limited range of deflection angles and upstream Mach numbers and are derived by the authors using the physically exact relations for a two-dimensional oblique shock wave in the form of Eq. (1), which was stated for the first time by Schubert[2] in 1943 as

$$\frac{\tan(\beta-\theta)}{\tan\beta}=\frac{(\gamma-1)M_1^2\sin^2\beta+2}{(\gamma+1)M_1^2\sin^2\beta} \tag{1}$$

The disadvantage of Eq. (1) is the implicit relation between θ, β, and M_1, which does not allow the wave angle β to be solved directly.

Nevertheless, the explicit and analytically exact expression for the wave angle β resulting from Eq. (1) was already derived in 1972 by the German mathematician Wellmann.[3] By rewriting the more general form of Eq. (1)

$$\frac{\sin(\beta+\alpha)\sin(\beta-\alpha)\cos(\beta-\theta)}{\sin\beta\sin\theta}=\frac{\gamma+1}{2} \tag{2}$$

$$\sin\alpha=1/M_1 \tag{3}$$

in a cubic relation for $\tan\beta$, Wellmann obtained the following equation:

$$\tan\theta\left[\frac{\gamma-1}{2}+\frac{\gamma+1}{2}\tan^2\alpha\right]\tan^3\beta-\tan^2\beta + \tan\theta\left[\frac{\gamma+1}{2}+\frac{\gamma+3}{2}\tan^2\alpha\right]\tan\beta+\tan^2\alpha=0 \tag{4}$$

The solution of Eq. (4) is easily obtained and reads

$$\tan\beta=\frac{b+9ac}{2(1-3ab)} - \frac{d(27a^2c+9ab-2)}{6a(1-3ab)}\cdot\tan\left[\frac{n}{3}\pi+\frac{1}{3}\arctan\frac{1}{d}\right] \tag{5}$$

$$n=0,\ 1,\ -1$$

with

$$a=\left[\frac{\gamma-1}{2}+\frac{\gamma+1}{2}\tan^2\alpha\right]\tan\theta$$

$$b=\left[\frac{\gamma+1}{2}+\frac{\gamma+3}{2}\tan^2\alpha\right]\tan\theta$$

$$c=\tan^2\alpha$$

$$d=\sqrt{\frac{4(1-3ab)^3}{(27a^2c+9ab-2)^2}-1}$$

From Eq. (5), the following solutions for the wave angle β of a two-dimensional oblique shock wave can be obtained:

$n=0$	the weak solution ($M_2 <> 1$)
$n=1$	the strong solution ($M_2<1$)
$n=-1$	a physically meaningless solution (decreasing entropy)

Since Eq. (5) represents the exact solution of Eq. (1) for the wave angle β of weak and strong oblique shocks in terms of the deflection angle θ and the upstream Mach number M_1, no approximations to Eq. (1) are necessary any more.

Finally, it should be noted that Wellmann also derived explicit expressions for the maximum wave angle θ_m (i.e., the limiting wave angle for which the shock is attached) and the sonic wave angle θ_s (i.e., the wave angle for which $M_2=1$). The solutions are

$$\cos 2\theta_m=\frac{1}{\gamma}\left[\left(\frac{\gamma+1}{2}-\cos 2\alpha\right) - \sqrt{\gamma+1}\cdot\sqrt{\left(\frac{\gamma+1}{2}-\cos 2\alpha\right)^2+\frac{\gamma}{4}(3-\gamma)}\right] \tag{6}$$

$$\sin^2\theta_s=\frac{1}{2\gamma}\left[\left(\frac{\gamma-3}{2}\sin^2\alpha+\frac{\gamma+1}{2}\right) + \sqrt{4\gamma\sin^4\alpha+\left(\frac{\gamma-3}{2}\sin^2\alpha+\frac{\gamma+1}{2}\right)^2}\right] \tag{7}$$

Furthermore, Wellmann derived a lot of useful explicit expressions for the evaluation of shock wave reflections at solid walls and freestream boundaries. For details see the original report, Ref. 3.

References

[1]Dou, H. S., and Teng H. Y., "Approximate Formula of Weak Oblique Shock Wave Angle," *AIAA Journal,* Vol. 30, No. 3, 1992, pp. 837–839.

[2]Schubert, F., "Zur Theorie des stationären Verdichtungsstoßes," *Zeitschrift für Angewandte Mathematik und Mechanik,* Vol. 23, 1943, pp. 129–138.

[3]Wellmann, J., "Vereinfachung von Rechnungen am schiefen Verdichtungsstoß," *Deutsche Luft- und Raumfahrt,* Forschungsbericht 72-11, Deutsche Forschungs- und Versuchsanstalt für Luft- und Raumfahrt, Institut für Aerodynamik, Braunschweig, Germany, Jan. 1972.

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